Alg2/Trig

Final Review – Ch3 Quadratic Equations and Complex Numbers

Name KEY

Period ______

Methods for Solving Quadratic Equations

Since there are many options for solving quadratic equations, give a short explanation as to when you would choose each method.

(1) Graphing Method when tool is available; when approximate answer is ok

(2) Using Square Roots when only an $(x-h)^2$ term w/o a x-term (i.e, b=0)

(3) Factoring when factorable

(4) Completing the Square anytime works, (5) is normally preferable if (2) or (3) aren't possible

(5) Quadratic Formula anytime works

Solve the following equations. Be sure to use a variety of methods.

1. $x^2+3x-4=0$

$(x+4)(x-1)=0$

$\boxed{x=-4 \quad x=1}$

2. $2x^2-8x-5=0$

$x=2\pm\frac{\sqrt{26}}{2}$

$\frac{8\pm\sqrt{64+40}}{4}$

3. $x^2+41=-8$

$x^2=-49$

$\boxed{x=\pm 7i}$

4. $x^2-6x=-8$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$\boxed{x=4 \quad x=2}$

5. $-2(x+5)^2=100$

$(x+5)^2=-50$

$x+5=\pm 5i\sqrt{2}$

$\boxed{x=-5\pm 5i\sqrt{2}}$

6. $2x^2-5x=0$

$x(2x-5)=0$

$\boxed{x=0 \quad x=\frac{5}{2}}$

7. $(2x-6)^2+4=16$

$(2x-6)^2=12$

$2x-6=\pm 2\sqrt{3}$

$\boxed{x=3\pm\sqrt{3}}$

8. $2x^2+3x=6x-10$

$2x^2-3x+10=0$

$x=\frac{3\pm\sqrt{9-80}}{4}$

$=\frac{3}{4}\pm\frac{i\sqrt{71}}{4}$

9. $ax^2+bx+c=0$

$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$

Types of Solutions to Quadratic Equations

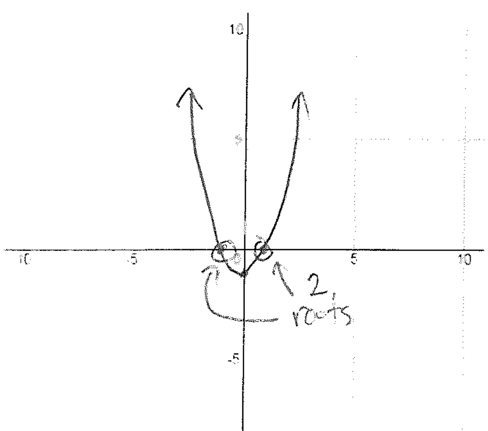
We are often taking a quadratic equation in x and y and substitution 0 in for y to find the roots of the equation. There are 3 possible outcomes. For each possible outcome listed below, give an example of a quadratic function, show how to find the roots, and show graphically what the solutions look like. Write each function in one of these forms:

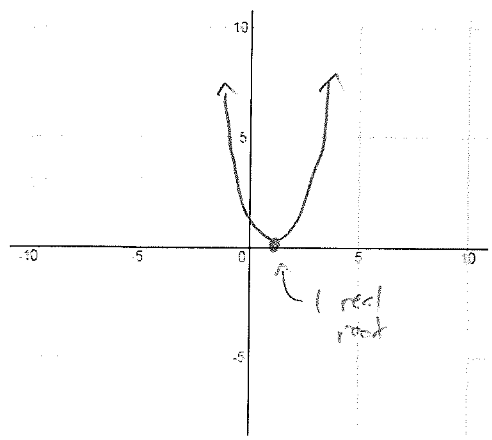
$y = ax^2 + bx + c \qquad y = a(x-h)^2 + k \qquad y = a(x-p)(x-q)$

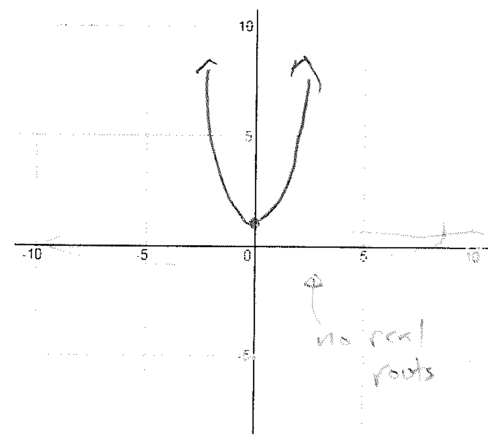
10. 2 real roots: $y = x^2 - 1$

11. 1 real double root: $Y = (x-1)^2$

12. 2 imaginary roots (0 real roots): $y = x^2 + 1$







Recall that the **discriminant** is the part under the square root in the Quadratic Formula. We can tell if the roots will be imaginary or real by looking at the sign of the discriminant. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Find the discriminant for each quadratic and state if the solutions are real or imaginary, and state how many of each.

13. $4x^2 - 24x = -36$

$x^2 - 6x + 9 = 0$

$b^2 - 4ac$

$36 - 36 = 0$ real

$\therefore \boxed{1 \text{ root}}$

14. $4x^2 - 6x + 9 = 0$

$b^2 - 4ac$

$36 - 4(36) < 0$

$\boxed{2 \text{ imag roots}}$

15. $\left(-2x = \frac{1}{2}x^2 - 6\right] \cdot 2$

$x^2 + 4x - 12$

$b^2 - 4ac$

$16 + 48 > 0$

$\therefore \boxed{2 \text{ real roots}}$

Sets of Numbers – Introduction to Imaginary Numbers

Write 2 examples of numbers for each set of numbers in the flow chart below.

Complex Numbers

Real Numbers

Imaginary Numbers

Rational Numbers

Irrational Numbers

Complex Numbers

Simplify the expression. Give your answer in standard form for a complex number.

16. $\sqrt{64} =$ $\underline{8}$

17. $\sqrt{-6} =$ $\underline{i\sqrt{6}}$

18. $(3+i)-(5-2i) =$ $\underline{-2+3i}$

19. $(3+2i)(4-5i)$

$12+8i-15i-10i^2$

$12-7i \quad +10$

$\boxed{22-7i}$

20. $(2i)^7$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^7 = -i$

$2^7(-i) = \boxed{-128i}$

21. $(6-2i^4)-(2-6i^3)-(i^{10})$

$-i \qquad -1$

$6-2-2+6(-i)-(-1)$

$= \boxed{3-6i}$

Solve the equation:

22. $2x-5i = 9+yi$

$\boxed{x = \frac{9}{2} \quad y = -5}$

23. $6+2i = x-yi$

$\boxed{x = 6 \quad y = -2}$

24. $x^2+4 = -21$

$x^2 = -25$

$\boxed{x = \pm 5i}$

25. $x(x+6) = -24$

$x^2+6x+24 = 0$

$x = \dfrac{-6 \pm \sqrt{36-4(24)}}{2}$

$= \boxed{-3 \pm i\sqrt{15}}$

$96 - 36 = 60$

$\sqrt{-60} = 2i\sqrt{15}$

26. $3x^2+8x = 2x-9$

$3x^2+6x+9 = 0$

$x^2+2x+3 = 0$

$x = \dfrac{-2 \pm \sqrt{4-12}}{2}$

$\boxed{= -1 \pm i\sqrt{2}}$

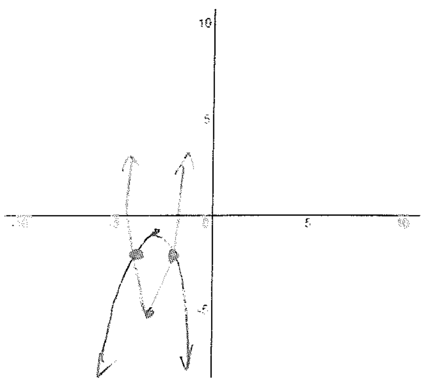
Systems of Equations

If a point is a solution to a system of equations, what must be true about that point?

---

27. Solve the following system of equations with each of the 3 methods. $\begin{cases} y=-x^2-6x-10 & ① \\ y=3x^2+18x+22 & ② \end{cases}$

Graphing Method:



Substitution Method:

$-x^2-6x-10 = 3x^2+18x+22$

$4x^2+24x+32=0$

$x^2+6x+8=0$

$(x+4)(x+2)=0$

$\boxed{x=-4 \quad x=-2}$

$y=-16+24-10 = -2$ $y=3(4)-36+22 = -2$

$\boxed{(-4,-2) \ \& \ (-2,-2)}$

Elimination Method:

subtract ① − ②

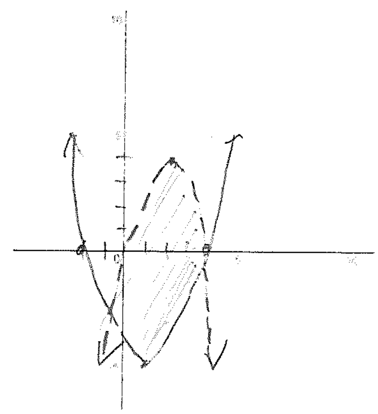
$0=-4x^2-24x-32$

$x^2+6x+8=0$

Systems of Inequalities and Inequalities with 1 Variable

When you have a system of inequalities, the solution will be represented graphically. Shade the solution to the system.

28. $\begin{cases} y<-(x-2)^2+4 \\ y\geq\frac{1}{2}(x+2)(x-4) \end{cases}$



When an inequality only has one variable, we can solve algebraically or graphically and represent the solution on a number line.

29. Given: $x^2-4x+1>0$

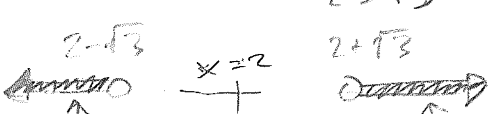
Solve algebraically: $x=\frac{4\pm\sqrt{16-4}}{2}$

$= 2\pm\sqrt{3}$

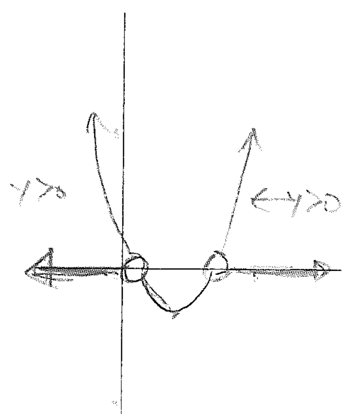
test $x=2$

$4-8+1 \not> 0$

∴



Solve graphically:



Modeling Problems

Projectile Motion: $h(t)=-16t^2+v_0t+h_0$ Note: If object is dropped then $v_0=0$.

30. A golfer hits the ball into the air. The ball is on a hill 12 feet above the landing area (or the fairway) and has an initial velocity of 128 feet per second. How long is the ball in the air before it lands on the fairway? ← h=0 on fairway

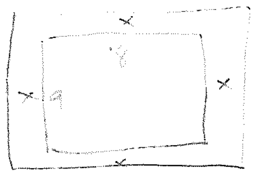
$h(t)=0=-16t^2+128t+12$

$4t^2-32t-3=0$

8.09 sec

$t=\frac{32\pm\sqrt{32^2+48}}{8} = 4\pm\frac{\sqrt{67}}{2} \approx \boxed{\text{8.09 sec}}$

31. Your friend's family has a rectangular pool that measures 18 yards by 9 yards. The family wishes to put a deck around the pool but they are not quite sure how wide to make the deck. Since you are an ace math student, you offer to your skills to help determine the deck width. If the total combined area of pool and deck is to be 400 square yards and the deck is to have uniform width, how wide should the deck be? Justify your answer with a diagram, properly labeled, and supported by your calculations.



$400 = (18+2x)(9+2x)$

$400 = 162 + 18x + 36x + 4x^2$

$4x^2 + 54x - 238 = 0$

$2x^2 + 27x - 119 = 0$

$x = \dfrac{-27 \pm \sqrt{27^2 + 8(119)}}{4}$

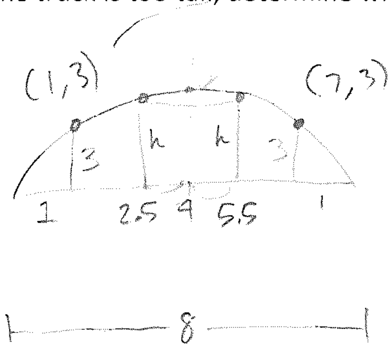
$= \dfrac{-27 \pm 41}{4}$

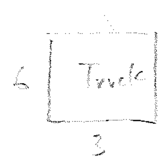
$= \dfrac{14}{4} \neq \cancel{-17}$

$= \boxed{\dfrac{7}{2}}$

$\boxed{x = 3.5 \text{ yards}}$

32. A tunnel, with a parabolic arch, is 8 mtrs wide. If the height of the arch 1 mtr from the outer edge of the arch is 3mtrs, can a truck that is 6 mtrs tall and 3 mtrs wide fit through the arch? Draw and label a diagram. Determine an equation that models the arch of the tunnel. Support your answer with the appropriate calculations. If you determine the truck is too tall, determine what truck width would support the height.





$y = a(x)(x-8)$

$3 = a(1)(1-8)$

$a = -\dfrac{3}{7}$

$y = -\dfrac{3}{7}x(x-8)$

when $x = 2.5$

$y = -\dfrac{3}{7}(2.5)(2.5-8)$

$\boxed{y = 5.8}$

Tunnel Equation: $y = -\dfrac{3}{7}x(x-8)$

Does the truck Fit: no since $5.8 < 6$ If no, what width of the truck would support the height

← 2.8 meters

$7 \cdot \left[6 = -\dfrac{3}{7}(x)(x-8)\right]$

$42 = -3x^2 + 24x$

$x^2 - 8x + 14 \quad 0$

$x = \dfrac{8 \pm \sqrt{64 - 4(14)}}{2}$

$= \dfrac{8 \pm 2\sqrt{2}}{2} = 4 \pm \sqrt{2}$

$\begin{array}{r} 14 \\ \times 4 \\ \hline 56 \end{array}$

width is $2 \cdot \sqrt{2} = \boxed{2.8 \text{ meters}}$

Solving quadratic inequalities in one variable.

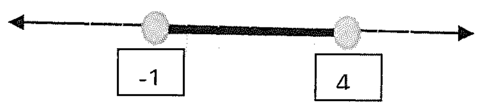
A quadratic inequality can be written in one of the following forms, where a, b, and c are real numbers and a does not equal zero.

$x^2+bx+c<0 \quad ax^2+bx+c>0 \quad ax^2+bx+c\le 0 \quad ax^2+bx+c\ge 0$

You can solve quadratic inequalities using algebraic methods or by using graphs.

Example: Solve $f(x)=x^2-3x-4<0$ algebraically. First, factor the equation and find the zeros. $(x-4)(x+1)=0$, $x=4 \;\; or \;\; x=-1$.

Next, plot the zeros on a number line and test the appropriate regions.



$f(-2)=(-)(-)=+ \quad f(0)=(-)(+)=- \quad f(5)=(+)(+)=+$ So, the solution is: $-1<x<4$.

test $x=0$ ✓ $x=10$ ✓

$4-\sqrt{14}$ $\quad 4+\sqrt{14}$

Solve the following inqualities:

33. $x^2+10x+9<0$

$(x+1)(x+\cancel{xx})<0$ $\quad x+9$ $\quad$ test $x=0$, $x=-2$

$+ \quad -9 \quad - \quad +$

~~xxxx~~ $\quad -1$

34. $x^2-11x\ge -28$

$x^2-11x+28\ge 0$ $\quad$ test $x=0$, $x=5$, $x=10$

$(x-7)(x-4)\ge 0$

$+ \quad - \quad +$

$4 \quad 7$

35. $\left[-\frac{1}{2}x^2+4x\le 1\right]\cdot -2$

$-x^2+8x\le 2$

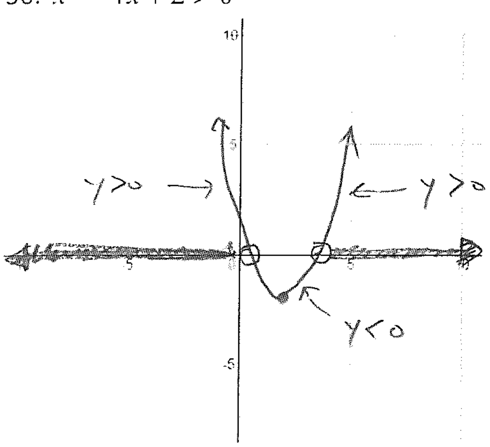
$x^2-8x+2\ge 0$

$x=\frac{8\pm\sqrt{64-8}}{2}$ $\quad \sqrt{56}=2\sqrt{14}$

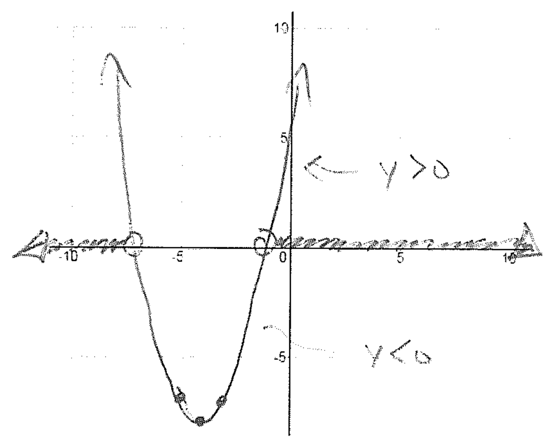
$=4\pm\sqrt{14}$

Solve the inequality by graphing:

36. $x^2-4x+2>0$



37. $x^2+8x>-7$



36. $(-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, +\infty)$

$x=\frac{4\pm\sqrt{16-8}}{2}$

$=2\pm\sqrt{2}$

$x_v=\frac{-b}{2a}=\frac{4}{2}=2$

$y=2^2-8+2$ $\quad$ vertex $(2,-2)$

$=4-8+2$

$=-2$

37. $(-\infty,-7)\cup(-1,+\infty)$

$x^2+8x+7>0$ $\quad (x+7)(x+1)>0$

$x=-7, x=-1$

$x_v=\frac{-b}{2a}=\frac{-8}{2}$ $\quad y=16-32+7$

$=-4 \quad = -9$

$(-4,-9)$